CVA, Premium or Charge? CVA Call Hedging

Christian Kamtchueng *

CTK.corp
London,UK

ESSEC
Paris,FRANCE

christian.kamtchueng@gmail.com

First version: Nov 20, 2010
This version: Jun 3, 2011

Abstract

After Lehman default (credit crisis 2007), Market participants started to consider the default risk as a major one. It became vital to charge the default risk at the trading level. The CVA became rapidly a standard when two institutions want to trade derivatives. The main task of this paper is to determine efficient hedging strategy of the CVA call. Indeed the litterature becomes to be hudge concerning this term without a clear definition. Is it a risk value or an actual premium?

*The opinions of this article are those of the author and do not reflect in any way the views or business of his employer.

Contents

1 Introduction

After Lehman Default, the financial industry began to consider the counterparty default risk as a major risk. The curve spread became not negligible, and the Credit Valuation Adjustment (CVA) and collateral agreement became very standard as protection against a default. "'No one is too big too default, every body owns a default risk.'"

In this context, most part of the exotic trades have been collateralised. The Market was so stressed that even in case of CSA, the industry was concerned by the residual remaining risk. The CVA became an usual practise to get rid of the credit risk. What is the CVA? It is a "'charge"' which is supposed taking into account the default of the seller of the derivative. By definition, the CVA is the difference between a default risky free price and the risky price. If we are buyer a derivative, we are long credit risk, to compensate this position, we agree with the derivative seller to be short CVA.

As a difference of premiums, is the CVA a price?

Premium of which optionality? This question seems very naive but even the experts don't have a clear answer. It is the first basic and ultimate question we should ask ourselves before going further computation.

The financial industry is full of abuses. Indeed a price should be linked to a hedging strategy. Most part of the CVA paper avoid this question in one hand, but they don't hesitate to called it "'premium"' in the other hand. Others preferred to called it "'charge"' and leaves the problem unsolved. The integral representation was a credit risk definition, which have a mathematical meaning in term of risk, therefore it is more a cost than a premium.

In [22], Kamtchueng proposed a methodology assuming the hedge of the CVA of vanilla call option. Boillier and Sorensen demonstrated a quasi perfect dynamic replication of the CVA swap thank to a strip of swaption (see [5]). Burgard and Kjaer described a dynamic hedging of the CVA in [18]. In [23], Jeanblanc and co formalised a markovian approach of the CVA modelling and hedging for a CDS.

Even if the integral representation of the CVA is correct in term of risk, the use of this as a trading price of the default optionality is not obvious at all.

$$CVA^{\Pi} \quad = \quad \mathbb{E}^{\mathbb{Q}}\left[\int_{t_0}^{T}(1-R)\,e^{-\int_{t_0}^{t} r_s ds}\Pi_t d\mathbb{Q}\left(\tau<t\right)\right]$$

People are talking about charges and benefits but is there an actual practical hedge for the CVA? The CVA should be seen as the premium of option to default if you consider it as a price. Most part of the experts have a credit risk background and they described it as a tradable price without established a clear and explicit hedging strategy. Piterbarg [19], Burgard and Kjaer [18] described a clear hedge via the PDE representation of the CVA. We will try to demonstrate the various hedging strategies proposed in the literature, innovate some of them and backtest them against Monte-Carlo simulation.

Our paper will be focus on the equity call option. It is a complement hedging focus to the previous works of Kamtchueng. Given the CVA hedging of the call, he proposed two different ways to price the default risky exotics.

First we will price theoretically the CVA call then we will established different hedging strategies to conclude on a PnL study in different contexts.

Focus on Equity, our work can be extended to any another asset classes.

Our results are the followings:

The dynamic hedging of the CVA expose us to different risk sources such as the discrete hedging, the variance of the potential payout (like an autocallable see [17]), the misspricing of some risks stochastic default intensity, forward credit and equity market.

More in favour of static hedging, we realised that this position insensitive to future market movement can be expensive, therefore a mixture seems to be more appropriate. By introducing new credit risk derivative structures, we allow the conversion of the credit risk into market risk. However we are still exposed to recovery risk and sensitive to the hedge seller default(defined as second order Wrong Way Risk in [21], which is not in the scope of our article).

2 Call Hedging

The hedging theory in CVA is not clear, indeed the CVA is related as we will explain later to the future market. In the rate market think to the liquidity of the swaption market, Boiller and Y established a hedging which tends to be perfect. How can we deal in the Equity market with the hedging of the CVA of a simple call?

2.1 Static Hedging

2.1.1 Long CCDS position

After 2008, the credit derivative market became very illiquid unlike the fear of default with became real. Some institutions start to propose some recovering services. They cover the default risk on derivatives via CCDS Contingent Credit Default Swap. It is an insurance which cover against the default risk of a counterparty regarding a predefined contingent. The problem in this type of product is the correlation between the hedge seller (short CCDS) and the counterparty (derivative seller). Indeed higher is the correlation useless is the protection. The second risk is more operational or settlement risk link to the derivative definition. More exotic is the derivative, more complex will be the price reconciliation.

We will ignore those two risks, however the last risk will be discussed in a subsequent publication. The product is an insurance against the default of an entity on a specific trade.

The buyer protection will give coupon until default or maturity of the trade whereas the seller agreed to cover the default of the entity regarding this specific derivative. For a call, the buyer protection will receive $(1-R)\,Call_t\,(T,K)$.

We will try to consider hedging strategies from the most naive to the most innovative one.

2.1.2 Long Weighted Call position

As a static hedging we can easily consider a static position on $(1-R)\,Call_t\,(T,K)$. It is clear that we will be able to deliver the call at default. But this long position strategy does not take into account any default risk. It is clearly an expensive replication. An acceptable consideration would be to invest to a default diversificator D^P: in case of default, the structure gives you the value at default of the derivative P, at maturity otherwise.

$$(1-R)\,D_t^{Call(T,K)} = (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\int_t^{T\wedge\tau} Call_s\,(T,K)\,ds\right]$$

We have to notice that we consider a deterministic recovery. Even if some works has be done in term of recovery risk (see for instance [16], [14] and [15]), even if we could use some uncertain world framework to estimate it. The structure can have a capped recovery at default and a deterministic at maturity.

$$\begin{aligned}(1-R)\,D_t^{Call(T,K)} &= \mathbb{E}^{\mathbb{Q}}\left[\int_t^{T\wedge\tau}(1-R^*)\,Call_s\,(T,K)\,ds\right]\\ R^* &= max\left(R,R^{realised}\right)1_{\{\tau<T\}}+R1_{\{\tau>T\}}\end{aligned}$$

The diversificator and the long position on the weighted call seems to have the same value however in case of default the payout can be less than $(1-R)\,Call_\tau\,(T,K)$. Therefore, if the seller believe on a default of the counterparty, it could price the cap on the recovery via a stochastic Recovery Model ([16]). For the buyer, it is a more attractive protection regarding our believe and utility to diversify.

2.2 Semi Static Strategies

In this section we will consider a discretisation of the time; $(T_i)_{0\le i\le N}$ with $T_0=t$ and $T_N=T$.

2.2.1 Long Strip of Forward Call Spread

Another way to replicate the CVA statically would be to consider a strip of forward starting option:

$$\begin{aligned}\check{C}VA^{Call} &= \mathbb{E}^{\mathbb{Q}}\left[\underbrace{\sum_{i=1}^{N}(1-R)\left[e^{-\int_t^{T_{i-1}}r_s ds}Call\,(T_{i-1},T,K)-e^{-\int_t^{T_i}r_s ds}Call\,(T_i,T,K)\right]}_{S_{1,N}}\right]\\ &= \sum_{i=1}^{N}(1-R)\left[\underbrace{FwCall_t\,(T_{i-1},T,K)-FwCall_t\,(T_i,T,K)}_{\psi^i}\right]\\ &= 0\end{aligned}$$

Our positions gives the ability to replicate the call value between each $\delta^i := [T_{i-1},T_i]$. A straight forward analysis of the cash flows, gives the following results for each ψ_i:

- When the default occurs during the period $\tau\in\delta^i := [T_{i-1},T_i]$ we give the $(1-R)\,Call_\tau\,(T,K)$ and sell the rest of the remaining structure which implies be short the call.
- Otherwise our long position will be cancelled at T_i.

In fact, it is easy to see that this strategy is equivalent to "‘wait and hope for the best"’, it is a proper bet on the non default event. Our result will show that the ridiculous can pay (in average)!

We can see the difference between the strategy proposed in the previous section based on a static position on $(1-R)\,Call\,(T,K)$ which generate a benefit in case of no default, unlike this one which generate losses in case of default (and delivering of the weighted call).

Both strategies are not very accurate because they are weakly related to the counterparty default. the first one long position on the weighted call do not really depends on the default of the counterparty. The second more sensitive will generate a loss which will increase with the probability of default.

The liquidity of the forward starting market could be another issue.

This strategy can be extended in function of the market view of the call seller. Indeed we could bet on a stock value lower than the strike in a future time T_i before maturity and a no default event before T_i, therefore we could invest on a forward starting option $FwdStartCallOption\,(t,T_i,T)$. This risky strategy can be cheaper than invest to a weighted call.

Still depending of the view of the seller, the correlation between the Equity and Credit Market can be considered In case of high correlation between the stock and the counterparty, the seller could decide to buy a Barrier option, which will be cheaper than the position on the weighted call.

Another strategy can be to invest in the weighted call in function of the market movement, for instance the call seller can wait $\tau^C := inf\left\{t\in[t_0,T]\,|Call_t\,(T,K)<\gamma\left(t_0,\tau^C\right)CVA_{t_0}\right\}$ by CVA_{t_0} we defined the CVA premium received from the counterparty (Call buyer). This strategy can be summarize by "'affordable protection"', given the premium received the call seller will be protect only is if the counterparty do not default before τ^C. In the scenarios presented in Figure 6-8, τ^C is before the maturity or the default. It is totally opposite to the pricing theory which associates a price to a hedge and not the other way wrong.

2.2.2 Long Strip of CDS

We have seen the profile of the call exposure which is sensitive to our forward skew. instead of buying forward starting contracts or options, we could consider the hedging of my call price dynamically. First we will described a pseudo semi static hedge based on a strip of CDS options

$$\begin{aligned}\check{C}VA^{Call} &= \mathbb{E}^{\mathbb{Q}}\left[\underbrace{\sum_{i=1}^{N}\left[CDS\left(T_i,T,\underbrace{N_{T_i}}_{Call^{BS}_{T_i}\left(T,K;\sigma_{t_0}(T-T_i,K)\right)}\right)-CDS\left(T_{i-1},T,N_{T_{i-1}}\right)\right]}_{S^{CDS}_{1,N}}\right] \\ &= \sum_{i=1}^{N}\left[FwCDS\left(T_i,T,N_{T_i}\right)-FwCDS\left(T_{i-1},T,N_{T_{i-1}}\right)\right]\end{aligned}$$

Given the following assumptions:

- the market is rolling over
- the stock is invariant

We can deliver $(1-R)\,Call\,(T,K)$ at default. Finer is the subdivision better is the hedge. Indeed we will be sensitive to a change of the market and a change of the stock value between δ^i.

By market rolling over we mean that the Implied Market stay the same for each tenors:

$$\begin{aligned}
\sum\nolimits_{t_0} &:= \left(\sigma_{t_0}\left(K_i, T_j\right)\right)_{0\leq i\leq N_K, 0\leq j\leq N_T} \\
\sum\nolimits_{t_l} &:= \left(\sigma_{t_l}\left(K_i, T_j\right)\right)_{0\leq i\leq N_K, 0\leq j\leq N_T} \\
\sum\nolimits_{t_0} = \sum\nolimits_{t_l} &\Rightarrow \forall 0\leq i\leq N_K, 0\leq j\leq N_T \\
&\quad\ \sigma_{t_0}\left(K_i, T_j\right) = \sigma_{t_l}\left(K_i, T_j\right)
\end{aligned}$$

How can we hedge the residual risk of our far to be perfect strategy, resulting of our strong assumptions?

In the next section, we will try to cover the residual risk not hedged by our strip of CDS in order to deliver $(1-R)\,Call_t\,(T,K)$ at default time using the OTC market or a dynamic hedging.

2.3 Dynamic Hedging

As explained previously, in order to hedge our long CVA position we could consider a hedging via a strip of CDS with the future call price as Notional. As the forward starting problem, the future price of the call is unknown. We have hedged the time value of the call but we are still sensitive to asset movement and implied vol movement.

2.3.1 Delta Risk

We will keep the Black and Scholes assumption about the deterministic market volatility structure of our call option and be focus on the hedging of the delta risk.

First, we consider a typical delta hedging

$$Call_T\left(T,K\right) - Call_0\left(T,K\right) = \mathbb{E}^{\mathbb{Q}}\left[\int_0^T \frac{\partial Call_t\left(T,K\right)}{\partial s} dS_t + \int_0^T r\left[Call_t\left(T,K\right) - \frac{\partial Call_t\left(T,K\right)}{\partial s} S_t\right] dt\right]$$

We are exposed to $(1-R)\,Call_\tau\,(T,K)$, therefore we will consider the dynamic hedging of the weighted call until default (again we suppose the recovery deterministic).

$$\begin{aligned}(1-R)\left[Call_{T\wedge\tau}(T,K)-Call_0(T,K)\right] &= (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\int_0^{T\wedge\tau}\frac{\partial Call_t(T,K)}{\partial s}dS_t\right]\\ &+ (1-R)\,\mathbb{E}^{\mathbb{Q}}\left[\int_0^{T\wedge\tau} r\left[Call_t(T,K)-\frac{\partial Call_t(T,K)}{\partial s}S_t\right]dt\right]\end{aligned}$$

It can be very strange and no intuitive to buy a call and ending up hedging one part of it.

Instead to delta hedge classically the weighted call, we can go to the OTC market :

- by buying a Equity Delta Return Swap

  An Equity Delta Return Swap $EDS_t\left(T,K,(T_i,\sigma_i)_{0\le i\le N}\right)$ is a structure which gives to the buyer at each fixing T_i, $\frac{\partial}{\partial s}C^{BS}_{T_i}(T,K;\sigma_i)\left(S_{T_{i+1}}-S_{T_i}\right)$.
- or invest in a Strip of Forward Delta Stock

  A Forward Delta $FD_t(T_0,T,K,\sigma)$ is a contract promising delta call of stocks $S_{T_0}\frac{\partial}{\partial s}C^{BS}_{T_0}(T,K;\sigma_i)$ at time T_0. It can be replicated by a position long $FwCall(T_0,T,K;\sigma)$ and $K\times FwDigital(T_0,T,K;\sigma)$.
- or consider a Margin Zero Coupon Swap

  A Margin Zero Coupon Swap $MZCS(t,T,Call(T,K))$ is a contract which gives to the buyer the margin of a liquid derivative over a certain period of time. The seller receive the fair market value of the derivative at beginning of the trade. He gives back the value of the option minus a spread expressed in faire value percentage of the derivative fixed at the beginning of the trade at time T which can be before the maturity of the derivative.

Those strategies will be described in Appendix 5.3.

We have still a bias resulting of our Black and Scholes volatility assumption. In order to cover our loss or gain, we proposed to invest in a portfolio of weighted variance swap ([9] and [7] see for more details) or in a First Order Stock Variation (see description in Appendix 5.4).

- Call Gamma Swap (a Gamma Swap with a Notional equal to the Gamma of a call) : $(1-R)\,CGS(T,K)$
- Gamma Swap with a Notional equal to the average Gamma of a call or a the maximum value of the Gamma during the entire life of the trade : $(1-R)\,\bar{\Gamma}GS(T)$

In fact this second order risk, can be also captured by a portfolio of Corridor Variance Swap (see for instance the works of P.Carr [6] and R.Lee [8]). Indeed we can discretise asset and time space in bucket, for each bucket $B^k:=[K_k,K_{k+1}]$ we can consider a centroid y_i^k at time t_i as $y_i^k:=\mathbb{E}^{\mathbb{Q}}\left[S_{t_i}|S_{t_i}\in B^k\right]$, we also consider $\Gamma_i^k:=\mathbb{E}^{\mathbb{Q}}\left[\frac{\partial^2}{\partial s^2}Call_{t_i}(S_{t_i},T,K)\,|S_{t_i}\in B^k\right]$.

We will take a long (and expensive) position on $\left(CorridorVS\left(t_i, t_{i+1}, S, B^k, \underbrace{N_k^i}_{(1-R)\left(y_i^k\right)^2\Gamma_i^k}\right)\right)_{1\leq i\leq N_t, 1\leq k\leq N_B}$.

It can be seen easily that:

$$\Pi = \sum_{1\leq i\leq N_t, 1\leq k\leq N_B} \left(CorridorVS\left(t_i, t_{i+1}, S, B^k, N_k^i\right)\right)$$

$$\Pi \overset{\delta_B, \delta_t \to 0}{\rightarrow} \int_0^T \Gamma_t S_t^2 \left(\sigma^2 - \sigma^{mkt^2}\right) dt$$

2.3.2 Total Dynamic Hedging

Another way to hedge the CVA on the call would be to consider a total dynamic hedging of my CVA position. The liquidity of the credit market could be an issue in this context. In [18], C.Burgard and co proposed a PDE to compute the Bilateral CVA. Focus on the unilateral CVA we could in the same classical way compute a PDE assuming a hedging portfolio of the default risky price composed by stock S_t, counterparty risky Bond B^C and cash then deduce the CVA from the difference between the default risky price and the default risk free one.

We can notice that we have the same liquidity issue with the risky bond market. If our result will be similar we will be focus on the CVA, we do not want to imply any own default hedging. Indeed a such self financing hedging portfolio is not clear and problematic in practice.

$$\begin{aligned} U\left(t, S_t, J_t^I\right) &= \Pi_t^U \\ \Pi_t^U &:= \alpha_t S_t + \alpha_t^I P_t^I + \beta_t \end{aligned}$$

β_t is the cash component of my hedging portfolio, we will distinguish two cases:

1. the first one consider a cash which is put into a risk free bank account r (could be considered as the EONIA) or borrowed from the treasury at rate r_F.

2. the second one consider another source of funding, via a repo transaction we borrow our stock position against a cash amount at the repo rate r_{R_S}. The same with the Bond of the counterparty r_I. Care must be taken about the repo transaction, indeed few of them allow a change of ownership during the transaction (eg for instance the dividends are not ours anymore).

$$\frac{\partial U}{\partial t} + \underbrace{\mu_t}_{r_{R_S}-q_t} S_t \frac{\partial U}{\partial s} + \sigma^2 S_t^2 \frac{\partial^2 U}{\partial s^2} + \lambda_I \left(1 - R_I\right) V_t^+ = \underbrace{r^I}_{r+\lambda_I} U$$
$$U_T = 0$$

by Feynman-Kac we retrieve the classical definition of the unilateral CVA

$$\begin{aligned} U_t &= \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \left(1 - R^I\right) e^{-\int_t^s r_v dv} V_s^+ \lambda_s^I e^{-\int_t^s \lambda_v^I dv} ds\right] \\ &= \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \left(1 - R^I\right) e^{-\int_t^s r_v dv} V_s^+ d\mathbb{Q}\left(\tau < s\right)\right] \end{aligned}$$

More details of the demonstration are in Appendix.

We made some assumptions

- P^I the risky bond used to hedge the default of the issuer has a repo rate close to the risk free
- P^I is a bond with 0 recovery, therefore λ_I could be interpreted as a default intensity.

2.4 Wrong Way Risk

We have established the CVA PDE, we would like to implement the hedging portfolio which takes into account the Wrong Way Risk. The Wrong Way Risk is defined as the loss resulting of the correlation between the derivative and the counterparty:

- linked directly to the derivative value
- related to the type and value of the (riksy) collateral

We assume that the 3 jumps to default are independent and can not occur at the same time. The institution G is not explicit and depends on the context, it could refer to a government bond or an artificial bond based on a sector index.

$$\frac{\partial U}{\partial t} + \underbrace{\mu_t}_{r_{R_S}+\lambda_S+\beta^S\lambda_G-q_t} S_t \frac{\partial U}{\partial s} + \sigma^2 S_t^2 \frac{\partial^2 U}{\partial s^2} + \underbrace{\bar{\lambda}_I}_{\lambda_I+\beta^I\lambda_G} \left(1 - R_I\right) V_t^+ = \left(r + \underbrace{\bar{\lambda}_I}_{\lambda_I+\beta^I\lambda_G} + \underbrace{\bar{\lambda}_S}_{\lambda_S+\beta^S\lambda_G} + \lambda_G\right) U$$
$$U_T = 0$$

3 World Simulation and P and L

3.1 World Simulation

In order to quantify the quality of the different strategies we will first of all consider a "'simulation of the world"' until a maturity T.

- the stock price $(S_t)_{0\leq t\leq T}$:

$$S_{t_i} = f^S\left(S_{t_{i-1}}; ...\right)$$

- the option market $(\sum_t)_{0\leq t\leq T}$:

$$\sum\nolimits_{t_i} = f^{\Sigma}\left(\sum\nolimits_{t_{i-1}}; ...\right)$$

- the rate $(r_t)_{0\leq t\leq T}$:

$$r_{t_i} = f^r\left(r_{t_{i-1}}\right)$$

- the default intensity $(\lambda_t)_{0\leq t\leq T}$:

$$\lambda_{t_i} = f^{\lambda}\left(\lambda_{t_{i-1}}\right)$$

- the default probabilities market vol $(\psi_t)_{0\leq t\leq T}$:

$$\psi_{t_i} = f^{\psi}\left(\psi_{t_{i-1}}\right)$$

thanks to the simulated risk factors, we have the opportunity to have at time t the price of the following hedging instruments:

- Stock price S_t
- Vanilla option prices $Call_t\left(T,K\right)$, $Put_t\left(T,K\right)$ and $VS_t\left(T\right)$ via $\sum_t$ and S_t
- Bond prices $B_t\left(T\right)$ via r_t
- Risky Bond $\bar{B}_t\left(T\right)$ via r_t, λ_t and ψ_t.
- Credit Default Swap $CDS_t\left(T\right)$ via λ_t and ψ_t (in fact $Pds_t\left(T\right) = f^{Pd}\left(\lambda_t, \psi_t, T\right)$, ψ represents a move of the default probability term structure)
- Default event via λ_t

3.2 Tests and Results

The tests are based on a classical CVA computation (scalable product one), the exposure has been computed then multiplied to a default probability structure.

$$
\begin{aligned}
CVA^{\Pi} &= \mathbb{E}^{\mathbb{Q}}\left[\int_{t_0}^{T}(1-R)\,e^{-\int_{t_0}^{t} r_s ds}\Pi_t d\mathbb{Q}\,(\tau<t)\right] \\
&\approx \sum_{i=1}^{N_t}\Pi_{t_i}\cdot\underbrace{\delta_i}_{\mathbb{Q}(\tau<t_i)-\mathbb{Q}(\tau<t_{i-1})} \\
&= E^{\Pi\mathbb{Q}}\otimes\delta^{\mathbb{Q}}
\end{aligned}
$$

We explicitly indicate the measure to differentiate the risk neutral framework (CVA related) and the historical or real measure (Credit Risk related). If we want to consider the wrong way risk we should compute the exposure conditional to the default of the counterparty, as a majority of market participant do not take into account the wrong way risk we decide to see how will our hedging react to the correlation between the counterparty intensity and the stock price value. In figure 3 and 4, we present our numerical computation of the CVA via the credit survival probability and the exposure of the call.

3.2.1 Strategy Descriptions

- Strategy 1: A position long on CCDS as an insurance we will benefit in case of default and loss in case of survival.
- Strategy 2: A long position on $(1-R)\,Call$, it is an expensive protection which is still sensitive to a recovery misspesification.
- Strategy 3: A long position on a diversificator, similar to Strategy 2 cheaper because of the cap protection against the recovery.
- Strategy 4: Unconventional, it is the "‘wait and hope for the best’" strategy. It could seem crazy to bet on a no default but it happens in practise.
- Strategy 5: A long position strip of CDS spread. It is a protection against the today expected exposure. If the stock and the market dont move the protection is perfect.
- Strategy 6: A long position strip of CDS spread (strategy 5) and a dynamic delta hedging of the weighted call. The CDS part is recovery insensitive which is not the case of the dynamic hedging part. A long position strip of CDS spread.
- Strategy 7: A pure dynamic hedging of the CVA

We do not incorporate the cost of carry and funding risk in our study, we use the same risk free rate. This subject will be discussed in a subsequent publication. Indeed as mentioned Piterbarg [19], Burgard [18] and Kamtchueng [22], the way the trading desk manage the hedging portfolio will imply a different cost of carry. Not only concerning the hedging strategy but the way the trading interact with its treasury, the strategy concerning its risky asset (used as collateral or not). Kamtchueng was focus on the so-called second order Wrong-Way risk, for each strategy we are sensitive to the default of the CVA hedging seller eg the correlation between its default and the one of the counterparty.

3.2.2 Tests Descriptions

- test 1 We used the pricing parameters
- test 2 The dynamic hedging is done via the pricing implied volatility and the pricing default term structure
- test 5 Realised Recovery lower than the pricing assumption (40% against 60%), stress of the stock volatility and stress of the default intensity volatility.
- test 6 Realised Recovery lower than the pricing assumption (40% against 60%), stress of the stock volatility and stress of the default intensity volatility with a dynamic hedging using the pricing parameter.
- test 26 Correlation Stock-Rate 30% and Stock-Intensity 30%.
- test 27 Correlation Stock-Rate 0.0% and Stock-Intensity -30%.

3.2.3 Test Results and Comments

	$\mathbb{E}^{\mathbb{Q}}\left[PnL_{\tau\wedge T}\right]$						
Tests	S1	S2	S3	S4	S5	S6	S7
1	-0.649	13.95	13.95	1.928	8.661	8.588	10.95
2	-0.727	14.84	14.84	2.645	9.270	9.076	13.26
5	1.621	-6.478	-6.478	-16.43	-1.988	-1.782	-10.59
6	1.610	-4.425	-4.425	-17.56	-3.070	-2.904	-17.63
25	-0.612	10.596	10.596	-1.166	5.561	6.126	10.38
26	-0.723	16.211	16.211	-21.13	8.743	8.746	14.74

Table 1: Impact Pure Hedging, Stressed Market, and Wrong Way Risk in the Expected PnL

In the Figure 1 and 2, we populate the Profit and Loss distribution of the Strategies described in the previous subsection.

- In the Figure 1, we can see that S1 is the closest to 0 strategy. As an over hedging S2 and S3 are more expensive than any CVA number. Indeed thoses strategies do not

	$\mathbb{E}^{\mathbb{Q}}\left[PnL_{\tau\wedge T}\vert\tau<T\right]$							Default
Tests	S1	S2	S3	S4	S5	S6	S7	Rate
1	5.680	-9.731	-9.731	-16.91	-5.994	-5.45	-19.13	48.02%
2	5.455	-9.747	-9.747	-15.24	-4.589	-4.019	-14.75	48.28%
5	6.128	-27.50	-27.50	-35.89	-15.17	-14.24	-35.35	64.52%
6	6.132	-28.11	-28.11	-37.71	-16.89	-16.31	-46.68	64.44%
25	5.655	-9.739	-9.739	-22.92	-12.10	-10.36	-17.89	48.48%
26	5.506	-9.747	-9.747	-16.40	-5.772	-5.106	-14.58	48.10%

Table 2: Impact Pure Hedging, Stressed Market and Wrong Way Risk in the Expected PnL given Default

Test 1	S1	S2	S3	S4	S5	S6	S7
Mean	-0.649	13.95	13.95	1.928	8.661	8.588	10.95
Var	57.38	16817.1	16817.1	6025.4	5977.8	5444.6	9407.2
Max	16.42	2612.5	2612.5	22.19	37.61	146.95	1399.6
Min	-7.038	-12.05	-12.05	-2827.7	-2827.4	-2747.4	-1391.6

Table 3: Dynamic Hedging With Future Market

Test 2	S1	S2	S3	S4	S5	S6	S7
Mean	-0.727	14.84	14.84	2.645	9.271	9.076	13.26
Var	56.38	18430.7	18430.7	4128.4	4068	3559	10495.3
Max	16.42	2971.	2971.	22.48	38.87	296.5	3283.4
Min	-7.104	-12.05	-12.05	-2416.6	-2416.6	-2260.5	-409.7

Table 4: Hedging with Pricing Parameters

Test 6	S1	S2	S3	S4	S5	S6	S7
Mean	1.610	-4.425	-4.425	-17.56	-3.070	-2.905	-17.63
Var	62.41	22136	22136	37984.5	37862	36057.4	29118.5
Max	16.42	4041.8	4041.8	24.72	51.16	151.85	1538.7
Min	-7.384	-3136.8	-3136.8	-9356.1	-9352.2	-9194.2	-7468.9

Table 5: Stressed Market

Test 26	S1	S2	S3	S4	S5	S6	S7
Mean	-0.612	10.59	10.59	-1.166	5.561	6.126	10.38
Var	57.46	12168.4	12168.4	5792.2	5728.5	4891.9	8074.9
Max	16.42	2756.1	2756.1	22.28	38.53	113.3	1473.9
Min	-7.037	-12.03	-12.03	-2654.6	-2652.4	-2470.6	-954.9

Table 6: Correlation Stock-Rate 0.3,Stock-Intensity 0.3 Wrong Way Risk Result

Test 27	S1	S2	S3	S4	S5	S6	S7
Mean	-0.723	16.21	16.21	2.135	8.743	8.746	14.74
Var	55.92	28077.1	28077.1	7309.5	7269.5	6482.1	13864.1
Max	16.42	3754.7	3754.7	22.58	38.82	243.9	3630.4
Min	-7.077	-12.06	-12.06	-4207.1	-4206.1	-3963.3	-2197.3

Table 7: Test 27 : Correlation Stock-Rate 0.,Stock-Intensity -0.3 Wrong Way Risk Result

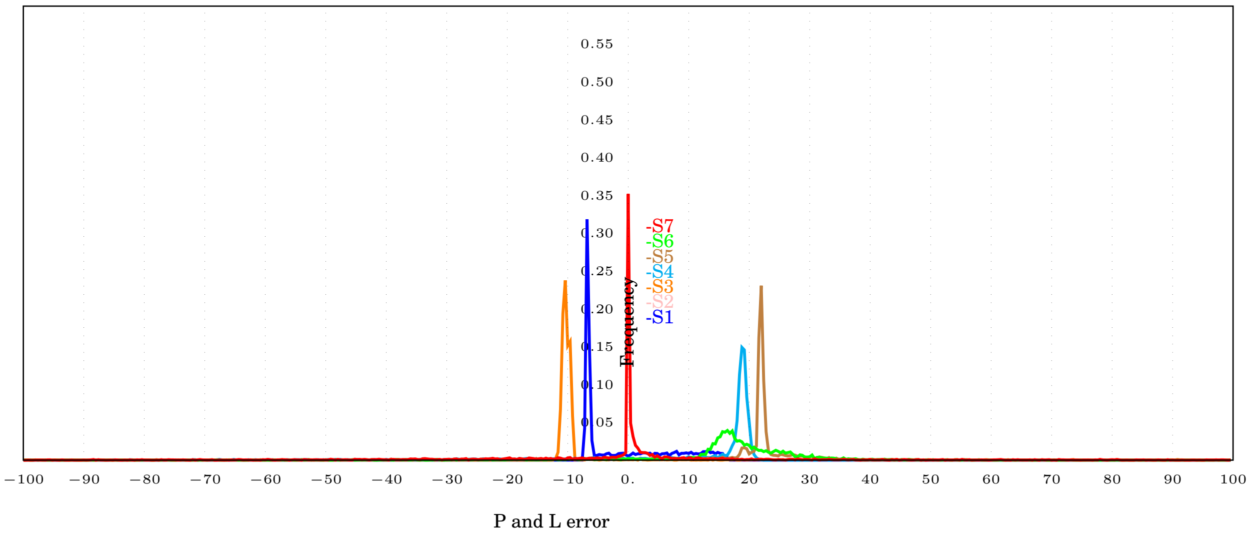

Figure 1: PnL simulation θ

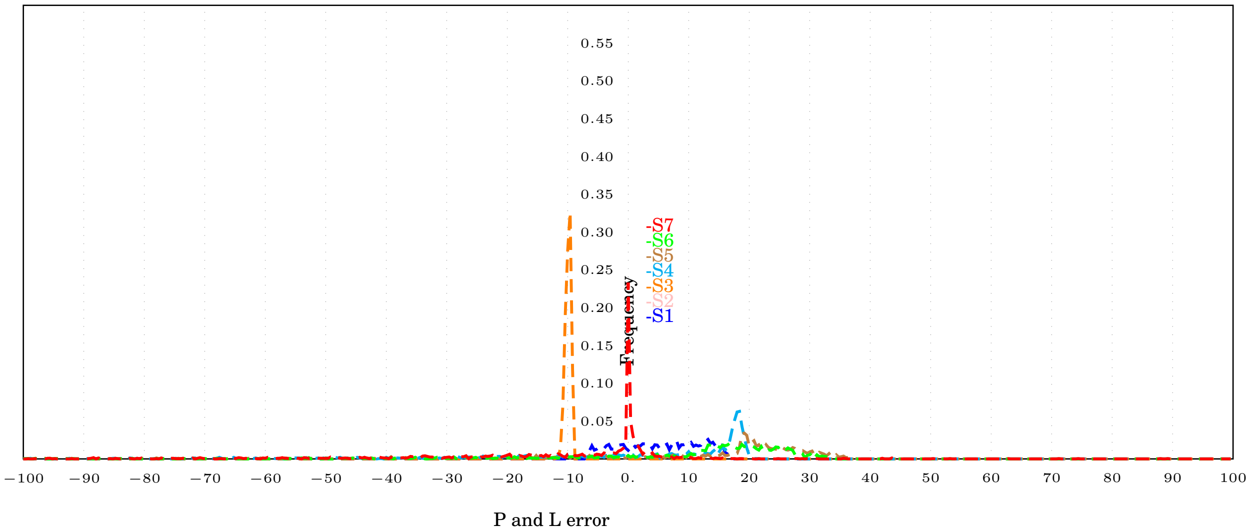

Figure 2: PnL given default simulation θ

take into account the counter party credit term structure. If in average the strategy S4 does not seem crazy we can see with the analysis of the distribution variance that we are exposed to huge losses. It can be unusual to see that our adjustment of S5 decreased our expected gains. As we explained with S4, our focus can not be on the expected gains but more on the control of our potential losses. In our study, we show for instance that in case of default the adjustment performs ($\mathbb{E}^{\mathbb{Q}}\left[PnL^{S5}_{T\wedge\tau}\right] > \mathbb{E}^{\mathbb{Q}}\left[PnL^{S6}_{T\wedge\tau}\right]$ but $\mathbb{E}^{\mathbb{Q}}\left[PnL^{S5}_{T\wedge\tau}|\tau < T\right] < \mathbb{E}^{\mathbb{Q}}\left[PnL^{S6}_{T\wedge\tau}|\tau < T\right]$)

- In the Figure 2, we can see that for S1 S4 S5 S6 and S7 the variance increases. As over replication strategies of a weighted call S2 and S3 have a same value when the counterparty default when the call is out of the money.

- In the Table 3, we can have more view on the distribution and potential risk of each strategy. The order of magnitude of S4 S5 and S6 is the same.

In Table 2, we studied the impact of a stress to the market and an overestimation of the recovery except S1 all the strategies have their expected PnL given default reduced. We also show the effect of the Wrong Way Risk, via the impact to an increase of the default intensity and the stock (which increase the payout in case of default).

In appendices, we provide the distribution of the PnL as the function of the default time.

- Figure 9, by the long position on CCDS, we give coupon against a potential default. So our losses increase with time.

- Figure 10, as an over replication strategy we never lose money and receive money whatever happens for S2. However if we over estimate the recovery, we are at risk when we enter in a Diversificator S3.

- Figure 11, We are losing money when the call value (or exposure) is in the money and higher than our carried premium. The losses can be huge.
- Figure 12-13, has noticed earlier, it seems that S5 and S6 are similar to S4. In figure 13 we show the difference between the strategies. The difference is positive or close to 0.
- Figure 14, the losses are reduced but still significant. We have a potential gains which is not visible in S4 S5 and S6.

4 Conclusion

We have presented a panel of Hedging Strategies of the CVA. The only theoretically perfect is the pure dynamic one S7, however the discrete hedging and the pricing assumption lead to very important potential losses in practices. We try to hedge an atomic event $\tau = t$ continuously in a discrete market. Concerning the CDS positions strategies (S5 and S6), they failed because of the strong modelling assumptions which do not take into account the move of the market and its correlation to the stock market. In fact, it seems that the major part of the risk is in the "'residual'". The cover of this residual can be done via a position on Variance Swap or Corridor Variance Swap. Therefore, this will increase our hedging portfolio price (mix of credit insurance, vanilla and OTC derivatives). It is clear after our study that the integral does cover our practical hedging cost.

Our works can be extended by :

- taking into account the transaction cost. This consideration could cancelled the dynamic hedging benefit
- considering the second order Wrong Way Risk. Some of those strategies are a pure credit risk transfer. Introduced by Kamtchueng, it is our sensitivity of our hedging seller default risk. Be long a CCDS to a counterparty more likely to default or 100% correlated to the counterparty is useless.
- taking into account the recovery risk. Transferring the credit risk to the market via vanilla or OTC derivatives expose us to an overestimation of the recovery.
- considering the error resulting of the discrete dynamic hedging of a supposed continuous strategy.

All those problems will be discussed in a subsequent publication. In [22], Kamtchueng assumed that the vanilla CVA is hedgable to introduce a new framework for the Derivative CVA portfolio. By establishing this assumption, we legitimate his approach.

More fundamental questions can be risen, regarding the calibration and our choice of hedging strategy. In this fear context, does the risk neutral framework still make sense? What are the impacts of our hedging CVA strategy on the Exposure computation (modeling and calibration)? We will answer to those questions in an ongoing book.

References

[1] F.Black and M.Scholes, *Pricing of Options and Corporate Liabilities*, **(1973)**,
the Journal of Political Economy

[2] R.C.Merton, *Theory of Rational Option Pricing*, **(1976)**,
Journal of Economics and Management Science: 141–183.

[3] R.C.Merton, *Options Pricing when underlying stock returns are discontinuous*, **(1976)**,
Massachussets Institute of Technology, Cambridge

[4] S.L.Heston, *A Closed-Form Solution for Options with Stochastic Volatility with Application to Bond and Currency Options*, **(1993)**
Review of Financial Studies 6 : 327-343.

[5] T.Boiller and E.Sorensen, *Pricing swap default risk*, **(1994)**
Financial Analysts Journal Vol. 50, No. 3: 23-33.

[6] P.Carr and D.Madan, *Towards a theory of volatility trading* , **(1998)**
Working Paper

[7] E.Derman, K.Demerterfi, M.Kamal and J.Zou, *More Than You Ever Wanted to Know About Volatility Swaps* , **(1999)**
Working Paper

[8] R.Lee, *Gamma Swap*, **(1999)**
Working Paper

[9] S.Bossu, E.Strasser and R.Guichard, *Just What You Need to Know about Variance Swaps* , **(2005)**
Working Paper

[10] D.Brigo and K. Chourdakis, *Counterparty Risk for Credit Default Swaps:Impact of Spread Volatility and Default Correlation*, **(2008)**
Working Paper

[11] D.Brigo and A.Pallavicini, *Counterparty Risk and Contingent CDS valuation under correlation between Interest Rates and Default*, **(2008)**
Risk Magazine

[12] D.Brigo and A. Capponi, *Bilateral Counterparty Risk Valuation with Stochastic Dynamical Models and Applications to Credit Default Swaps*, **(2009)**
Working Paper

[13] J.Gregory, *Being two-faced over counterparty credit risk*, **(2009)**
Risk Magazine

[14] M.Krekel, *Pricing distressed CDOs with Base Correlation and Stochastic Recovery*, **(2008)**

[15] S.Amroui and S. Hitier *Optimal Stochastic Recovery for Base Correlation*, **(2008)**
working paper

[16] C.Ech-Chabtbi, *CDS and CDO Pricing with Stochastic Recovery*, **(2008)**
working paper

[17] C.Kamtchueng, *Autocallable More American than European?*, **(2010)**
Working Paper

[18] C.Burgard and M.Kjaer, *PDE representations of options with bilateral counterparty risk and funding cost*, **(2010)**
Risk Magazine

[19] V.Piterbarg, *Funding beyond discounting collateral agreements and derivatives pricing*, **(2010)**
Risk Magazine

[20] Y.Elouerkhaoui, *Trading CVA: A new development in correlation modelling*, **(2010)**
Working Paper

[21] C.Kamtchueng, *CVA Implied Vol and Netting Arbitrage, Introduction*, **(2010)**
Working Paper

[22] C.Kamtchueng, *CVA Implied Vol and Netting Arbitrage*, **(2010)**
Working Paper

[23] T.R.Bielecki, S.Crepey, M.Jeanblanc and B.Zargari, *Valuation and Hedging of CDS Counterparty Exposure in Markov Copula Model*, **(2011)**
Working Paper

5 Appendices

5.1 CVA PDE's

5.1.1 Unilateral CVA PDEs

The demonstration is classic applying Ito Lemma to the price and computing the dynamic of the hedging portfolio. By identification of the martingal and finite variation parts, we determine the PDE satisfy by the CVA price.

The crucial point in a default fear world is the hedging portfolio. We could classically put the rest of our no risky asset in a bank account so we will have an additional gain via the stock dividends.

$$\begin{aligned} d\Pi_t^U &:= \alpha_t dS_t + \alpha_t^I dP_t^I + d\beta_t \\ &= \alpha_t dS_t + \alpha_t^I P_t^I \left(r_I dt - dJ_t^I \right) \\ &+ \left[r_t \left(\Pi_t^U - \alpha_t S_t - \alpha_t^I P_t^I \right)^+ + r_F \left(\Pi_t^U - \alpha_t S_t - \alpha_t^I P_t^I \right)^- + \alpha_t q_t S \right] dt \end{aligned}$$

You can enter in a repo transaction which allow you to borrow cash at rate r_R for the stock and r_R^I for the issuer bond against the deposit of your asset as collateral.

$$\begin{aligned} d\Pi_t^U &:= \alpha_t dS_t + \alpha_t^I dP_t^I + d\beta_t \\ &= \alpha_t dS_t + \alpha_t^I P_t^I \left(r_I dt - dJ_t^I \right) \\ &+ \left[r_t \left(\Pi_t^U \right)^+ + r_F \left(\Pi_t^U \right)^- - \alpha_t \underbrace{\mu_t}_{r_{R_S} - q_t} S - \alpha_t^I r_t P_t^I \right] dt \end{aligned}$$

At default we have $\Delta^I U_t = (1 - R) V_t^+ - U_t$.

$$\begin{aligned} dU_t &= \frac{\partial U}{\partial t} dt + \frac{\partial U}{\partial s} dS_t + \frac{1}{2} \frac{\partial^2 U}{\partial s^2} \sigma^2 S_t^2 dt + \Delta^I U dJ_t^I \\ \Delta^I U_t &= (1 - R) V_t^+ - U_t \end{aligned}$$

By identification of the martingale part and the finite variation one, we have :

- $\alpha_t = \frac{\partial U}{\partial s}$
- $\alpha_t^I = -\frac{\Delta^I U_t}{P_t^I}$

5.1.2 Wrong Way Risk CVA PDE

$$
\begin{aligned}
d\Pi_t^U &:= \alpha_t dS_t + \alpha_t^I d\tilde{P}_t^I \\
&+ \alpha_t^S d\tilde{P}_t^S + \alpha_t^G dP_t^G + d\beta_t \\
&= \alpha_t dS_t + \alpha_t^I \tilde{P}_t^I \left(r_I dt \underbrace{-dJ_t^I - \beta^I dJ_t^G}_{-d\tilde{J}_t^I} \right) \\
&+ \alpha_t^S \tilde{P}_t^S \left(r_S dt \underbrace{-dJ_t^S - \beta^S dJ_t^G}_{-d\tilde{J}_t^S} \right) + \alpha_t^G P_t^G \left(r_G dt - dJ_t^G \right) \\
&+ \left[r_t \left(\Pi_t^U - \alpha_t S_t - \alpha_t^I P_t^I - \alpha_t^S P_t^S - \alpha_t^G P_t^G \right)^+ + r_F \left(\Pi_t^U - \alpha_t S_t - \alpha_t^I P_t^I - \alpha_t^S P_t^S - \alpha_t^G P_t^G \right)^- + \alpha_t q_t S \right] dt
\end{aligned}
$$

if you use the securities to borrow money at the repo rate we have:

$$
\begin{aligned}
d\Pi_t^U &:= \alpha_t dS_t + \alpha_t^I d\tilde{P}_t^I \\
&+ \alpha_t^S d\tilde{P}_t^S + \alpha_t^G dP_t^G + d\beta_t \\
&= \alpha_t dS_t + \alpha_t^I P_t^I \left(r_I dt - dJ_t^I \right) + \alpha_t^I \tilde{P}_t^I \left(r_I dt - dJ_t^I - \beta^I dJ_t^G \right) \\
&+ \alpha_t^S \tilde{P}_t^S \left(r_S dt - dJ_t^S - \beta^S dJ_t^G \right) + \alpha_t^G P_t^G \left(r_G dt - dJ_t^G \right) \\
&+ \left[r_t \left(\Pi_t^U \right)^+ + r_F \left(\Pi_t^U \right)^- - \alpha_t \mu_t S_t - \alpha_t^I r_I \tilde{P}_t^I - \alpha_t^S r_S \tilde{P}_t^S - \alpha_t^G r_G P_t^G \right] dt
\end{aligned}
$$

we suppose our jumps to default are indepedents and we have at default:

- $\Delta^I U := (1-R) V^+ - U = (1-R)\, Call\,(T, K) - U$
- $\Delta^S U := 0 - U$
- $\Delta^G U := 0 - U$

In this framework we have two ways of defaulting either it is a idyiosincratic risk or a systemic via the J^G

By identification of the martingale part and the finite variation one, we have :

- $\alpha_t = \frac{\partial U}{\partial s}$
- $\alpha_t^I = -\frac{\Delta^I U_t}{\tilde{P}_t^I}$
- $\alpha_t^S = -\frac{\Delta^S U_t}{\tilde{P}_t^S}$
- $\alpha_t^G P_t^G + \alpha_t^I \beta^I \tilde{P}_t^I + \alpha_t^S \beta^S \tilde{P}_t^S = -\Delta^G U_t$

this is the general PDE for all positive option on stock

$$\frac{\partial U}{\partial t} + \underbrace{\mu_t}_{r_{R_S}+\bar{\lambda}_S-q_t} S_t \frac{\partial U}{\partial s} + \sigma^2 S_t^2 \frac{\partial^2 U}{\partial s^2} + (r - r_I)\, \Delta^I U + (r - r_S)\, \Delta^S U + (r - r_G)\, \Delta^G U = rU$$

$$U_T = 0$$

with $\bar{\lambda}_* = \lambda_* + \beta^* \lambda_G$ and $* \in \{S, I\}$.

5.2 World Simulation

- the stock price $(S_t)_{0 \leq t \leq T}$:

$$\frac{dS_t}{S_t} = r_t dt + \sigma^S dW_t^S$$

- the option market $(\sum_t)_{0 \leq t \leq T}$:

$$\sum\nolimits_{t_i} = \sum\nolimits_{t_0}$$

- the rate $(r_t)_{0 \leq t \leq T}$:

$$dr_t = \eta^r \left(\theta^r - r_t\right) dt + \psi^r dW_t^r$$

- the default intensity $(\lambda_t)_{0 \leq t \leq T}$:

$$d\lambda_t = \eta^\lambda \left(\theta^\lambda - \lambda_t\right) dt + \psi^\lambda \sqrt{\lambda_t} dW_t^\lambda$$

- the default intensity volatility $(\psi_t)_{0 \leq t \leq T}$:

$$d\psi_t = \eta^\psi \left(\theta^\psi - \psi_t\right) dt + \sigma^\psi \sqrt{\psi_t} dW_t^\psi$$

5.3 Credit Risk Structuring Products

- Forward Delta Call $FD_t\,(T_i, T, K, \sigma)$: it is a contract which gives to the buyer delta Stock at time T_i. Delta Black and Scholes with the forward market implied volatility. It is equivalent to a position on $FwCall\,(T_i, T, K)$ and $K \times FwDigital\,(T_i, T, K)$.

  business need:

  - market institution which wants to reduce his exposure to a particular futur date by buying delta stock.
  - traders or asset managers which have a view on the market and are not interesting to hold the stock during this period.

- Equity Delta Return Swap $EDS_t\left(T, K, (T_i, \sigma_i)_{0\leq i\leq N}\right)$: is a structure which gives you $\underbrace{\Delta_i}_{\frac{\partial}{\partial s}C_{T_i}(T,K)}\left(S_{T_{i+1}} - S_{T_i}\right)$. This structure can be replicated by investing $S_{T_i}\Delta_i$ at time T_i thanks to a loan, holding the long stock position until T_{i+1}.

  We have a bias resulting of the cost of funding: $\Delta_i\left(S_{T_{i+1}} - S_{T_i}e^{-\int_{T_i}^{T_{i+1}} r_s^f ds}\right)$

  business need: institution who wants to protect itself against a call movement.

- Margin Zero Coupon Swap and Margin Option on Derivative P
  - Margin Zero Coupon Swap $MZCS\,(t, T, P)$: the structure gives to the buyer a Margin on specific derivative P, the derivative has to be liquid enough to avoid any pricing consensus issue. Against an amount of derivative αP_{t_0}, the buyer receives frequently determined mark to market variation of the derivative. At maturity of the trade (which is not necessary the maturity of the derivative), the buyer retrieve αP_T.
  - Margin Call(Put) Option $MCO\,(t, T, P)$: this option gives to the buyer the positive (negative) Margin on specific derivative P.

  If the remuneration of the option is a premium given at the beginning of the trade, for the zero coupon swap there are many possible ways to remunerate the seller. As a Repo or Reverse Repo, the rate will depends of the market context.

  We have the spread value which can be determined as follows
  - In function of the future value of the derivative (expressed as a percentage of the future value)
  - At the beginning of the trade reducing the amount invest in derivative.
  - At each marginal delivering as a transaction cost
  - Or a combination of the previous ones.

  business need:
  - An investor who wants to protect himself against a market risk will be interested to invest on the option, as a buyer derivative with an unilateral collateral agreement which does not allow him to benefit of the positive margin call.
  - An institution which wants to cancel his collateral position or replicate the market variation of a specific derivative can be interested by a long position on Marginal Zero Coupon Swap.

- Diversificator: it is an option semi European which has an early payout event in case of default. If no default happens, the payoff is paid at maturity.

  business need: An investor who wants to protect himself against a default of an entity. In addition to the default protection, he can manage his recovery exposure via this type of contract.

We can easily imagine a spread diversificator which protects you on a part of the recovery. We can build a tranches market to allows investors to buy recovery protection in function of their needs. This market will need accurate recovery models in order to perform the pricing of those products.

5.4 First Order Stock Variation

the product is similar to a variance swap, the payoff is the following:

$$\Psi = \frac{N}{N}\sum_{i=1}^{N}\left(S_{t_i} - S_{t_{i-1}}\right)^2$$

we can replicate it via the sqare contract
Supposing our stock as an exponential martingale and the δ_i small enough

$$\begin{aligned}\Psi &\cong < S >_T \\ &= \int_0^T \sigma_t^2 S_t^2 dt\end{aligned}$$

applying the ito formula to the square contract and using a classical replication formula we have

$$\begin{aligned}\int_0^T \sigma_t^2 S_t^2 dt &= \left[S_0^2 - S_T^2 + \int_0^T \sigma_t S_t dW_t\right] \\ S_T^2 &= S_0^2 + 2S_0\left[S_T - S_0\right] \\ &+ 2\int_0^{S_0} (K - S_T)^+ dK + 2\int_{S_0}^{\infty} (S_T - K)^+ dK\end{aligned}$$

So by taking the expectation, we have the following price for a square contract:

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^T \sigma_t^2 S_t^2 dt\right] = \left[2\int_0^{S_0} Put\left(K,T\right) dK + 2\int_{S_0}^{\infty} Call\left(K,T\right) dK\right]$$

We have to notice that we have use the martingale property of the stock and supposed that the stochastic integral is bounded.

5.5 Strategies and PnL

- Static

- CCDS
  - At Default

$$\begin{aligned} PnL_{\tau} &= \gamma^{+}\left(t_0,\tau\right)CVA_{t_0}^{Call} - \gamma^{-}\left(t_0,\tau\right)\Pi_{t_0} \\ &- \underbrace{PremiumLeg^{CCDS^{Call}}\left(t_0,\tau\right)}_{\sum_{i=1}^{\eta(\tau)}\gamma^{-}(T_i,\tau)s} + (1-R)\,Call_{\tau}\left(T-\tau,K\right) \end{aligned}$$

  - At Maturity

$$\begin{aligned} PnL_{T} &= \gamma^{+}\left(t_0,T\right)CVA_{t_0}^{Call} - \gamma^{-}\left(t_0,T\right)\Pi_{t_0} \\ &- \underbrace{PremiumLeg^{CCDS^{Call}}\left(t_0,T\right)}_{\sum_{i=1}^{\eta(\tau)}\gamma^{-}(T_i,T)s} \end{aligned}$$

- Diversificator
  - At Default

$$\begin{aligned} PnL_{\tau} &= \gamma^{+}\left(t_0,\tau\right)CVA_{t_0}^{Call} - \gamma^{-}\left(t_0,\tau\right)\overbrace{\Pi_{t_0}}^{Div_{t_0}^{Call}(t_0,T,R,Ccy)} \\ &+ (1-R)\,Call_{\tau}\left(T-\tau,K\right) \end{aligned}$$

  - At Maturity

$$\begin{aligned} PnL_{T} &= \gamma^{+}\left(t_0,T\right)CVA_{t_0}^{Call} - \gamma^{-}\left(t_0,T\right)Div_{t_0}^{Call}\left(t_0,T,R,Ccy\right) \\ &+ (1-R)\left(S_T-K\right)^{+} \end{aligned}$$

- Semi Static
  - Forward Strip of FwdCall
    - At Default

$$\begin{aligned} PnL_{\tau} &= \gamma^{+}\left(t_0,\tau\right)CVA_{t_0}^{Call} - \gamma^{-}\left(t_0,\tau\right)\overbrace{\Pi_{t_0}}^{0} \\ &- (1-R)\,Call_{\tau}\left(T-\tau,K\right) \end{aligned}$$

    - At Maturity

$$PnL_{T} = \gamma^{+}\left(t_0,T\right)CVA_{t_0}^{Call}$$

  - Strip of FwdCDS

* At Default

$$
\begin{aligned}
PnL_\tau &= \gamma^+(t_0,\tau)\,CVA_{t_0}^{Call} - \gamma^-(t_0,\tau)\,\overbrace{\Pi_{t_0}}^{0} \\
&+ (1-R)\,N^{\eta(\tau)} \\
&- \sum_{i=0}^{\eta(\tau)} PremiumLeg^{CDS^{N^{i+1}}(T_{i+1},T)}(T_{i+1},\delta_i,\tau) - PremiumLeg^{CDS^{N^i}(T_i,T)}(T_i,\delta_i,\tau) \\
&- (1-R)\,Call_\tau(T-\tau,K)
\end{aligned}
$$

* At Maturity

$$
\begin{aligned}
PnL_T &= \gamma^+(t_0,T)\,CVA_{t_0}^{Call} \\
&- \sum_{i=0}^{N} PremiumLeg^{CDS^{N^{i+1}}(T_{i+1},T)}(T_{i+1},\delta_i,\tau) - PremiumLeg^{CDS^{N^i}(T_i,T)}(T_i,\delta_i,\tau)
\end{aligned}
$$

- Dynamic
  - Strip of FwdCDS and Delta Hedging
    * At Default

$$
\begin{aligned}
PnL_\tau &= \gamma^+(t_0,\tau)\,CVA_{t_0}^{Call} - \gamma^-(t_0,\tau)\,\overbrace{\Pi_{t_0}}^{(1-R)Call_{t_0}(T,K)} \\
&+ (1-R)\,N^{\eta(\tau)} \\
&- \sum_{i=0}^{\eta(\tau)} PremiumLeg^{CDS^{N^{i+1}}(T_{i+1},T)}(T_{i+1},\delta_i,\tau) - PremiumLeg^{CDS^{N^i}(T_i,T)}(T_i,\delta_i,\tau) \\
&+ (1-R)\left[\Delta_\tau^{BS} S_\tau - \underbrace{\beta_\tau^{BS}}_{KDigital^{BS}(T-\tau,K)}\right] \\
&- (1-R)\,Call_\tau(T-\tau,K)
\end{aligned}
$$

    * At Maturity

$$
\begin{aligned}
PnL_T &= \gamma^+(t_0,T)\,CVA_{t_0}^{Call} - \gamma^-(t_0,T)\,(1-R)\,Call_{t_0}(T,K) \\
&- \sum_{i=0}^{N} PremiumLeg^{CDS^{N^{i+1}}(T_{i+1},T)}(T_{i+1},\delta_i,\tau) - PremiumLeg^{CDS^{N^i}(T_i,T)}(T_i,\delta_i,\tau) \\
&+ (1-R)\left[\Delta_T^{BS} S_T - \beta_T^{BS}\right]
\end{aligned}
$$

  - Dynamic Hedging

* At Default

$$
\begin{aligned}
PnL_T &= \gamma^+(t_0,\tau)\,CVA^{Call}_{t_0} - \gamma^-(t_0,\tau)\overbrace{\Pi_{t_0}}^{\alpha_{t_0}S_{t_0}+\alpha^I_{t_0}P^I_{t_0}+\beta_{t_0}} \\
&- (1-R)\,Call_\tau(T-\tau,K) \\
&+ \alpha_\tau S_\tau + \alpha^I_\tau P^I_\tau + \beta_\tau
\end{aligned}
$$

* At Maturity

$$
\begin{aligned}
PnL_\tau &= \gamma^+(t_0,T)\,CVA^{Call}_{t_0} - \gamma^-(t_0,T)\,[\,\alpha_{t_0}S_{t_0} + \alpha^I_{t_0}P^I_{t_0} + \beta_{t_0}] \\
&+ \alpha_T S_T + \alpha^I_T P^I_T + \beta_T
\end{aligned}
$$

We have represented the cost of funding and the cost of carry via $\gamma^*, * \in \{+,-\}$. In our numerical result we consider $\gamma^+ = \gamma^-$ supposed equal to the risk free rate (if this terminology still makes sense). As explained V.Piterbarg, our pricing/hedging is dependant of our trading strategy: one can decide to use the risky assets (stock or risky bond) as collateral to another source of funding.

5.6 Numerical Test

Our numerical application we used 5000paths

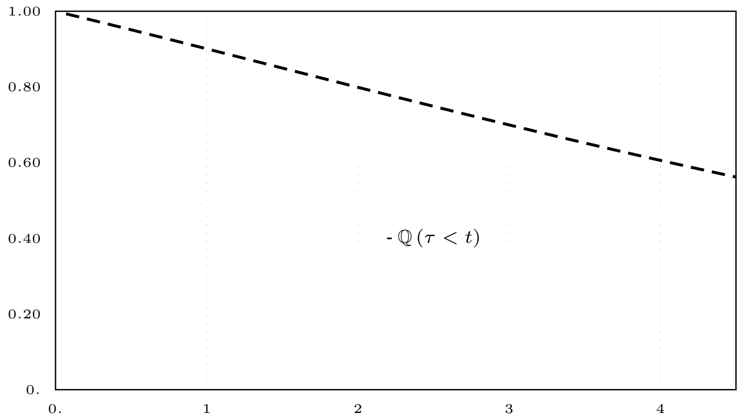

Figure 3: Credit Term Structure

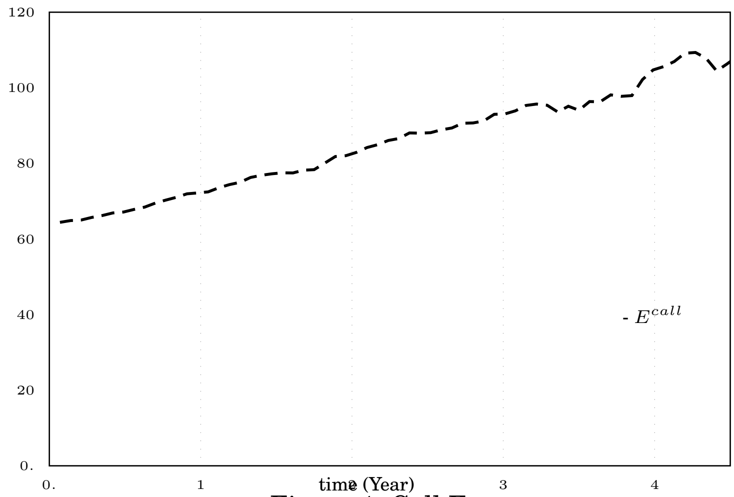

Figure 4: Call Exposure

5.7 Scenarios

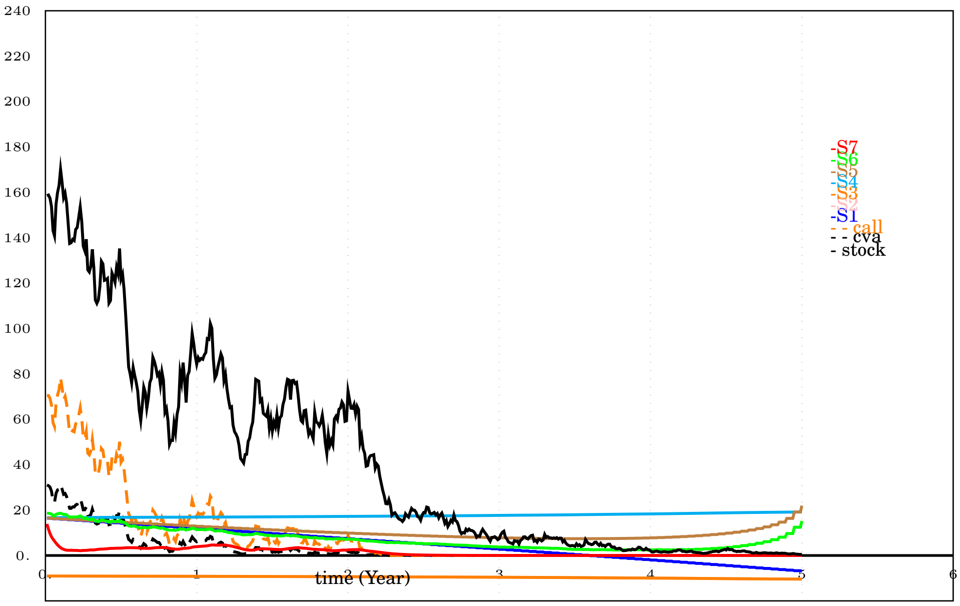

Figure 5: No Defaulting Scenario : test 1

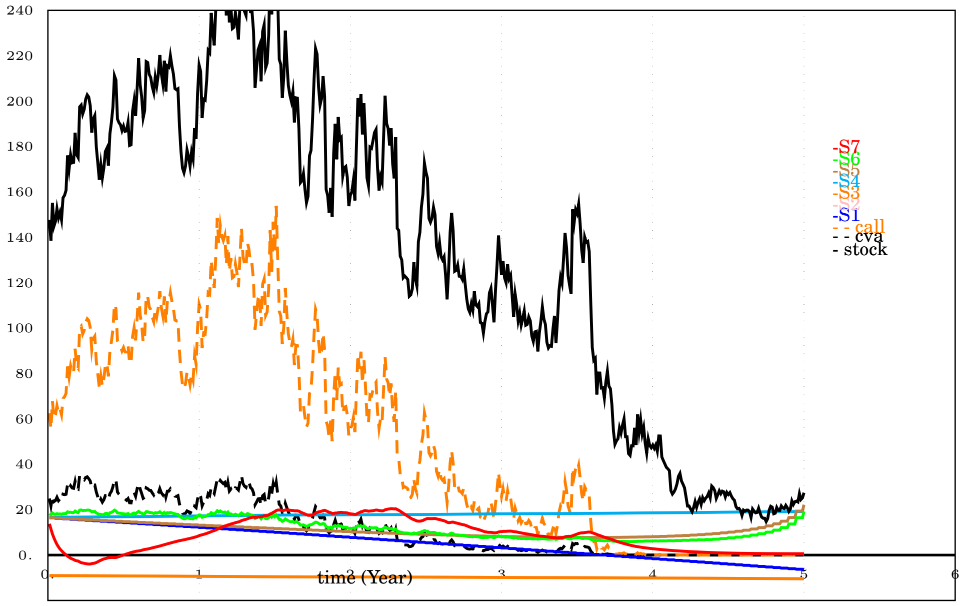

Figure 6: No Defaulting Scenario : test 2

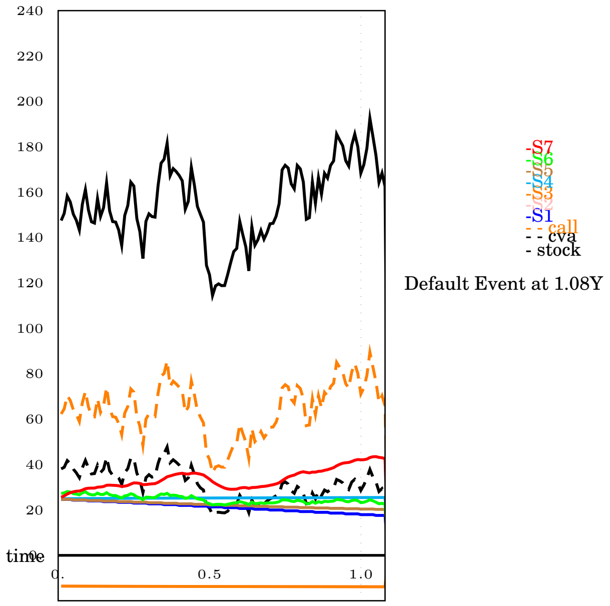

Figure 7: Defaulting Scenario : test 15

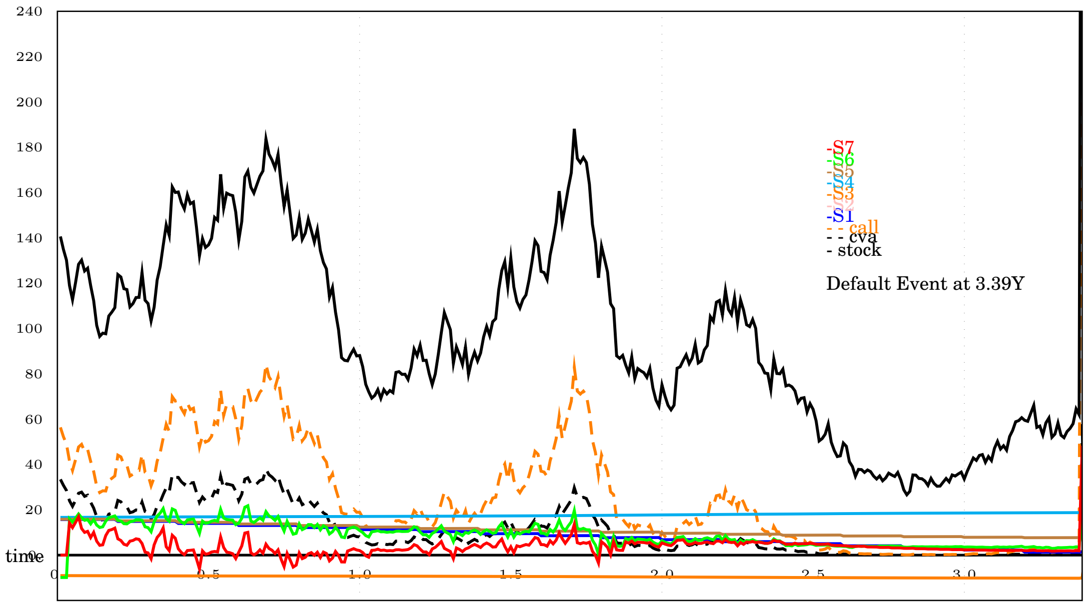

Figure 8: Defaulting Scenario : test 11

5.8 Strategy PnL as function of the time to default

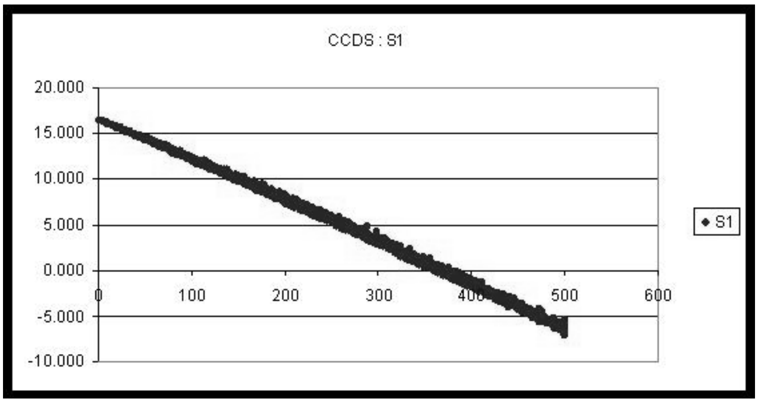

Figure 9: Strat 1 : CCDS

5.8.1 Strat1

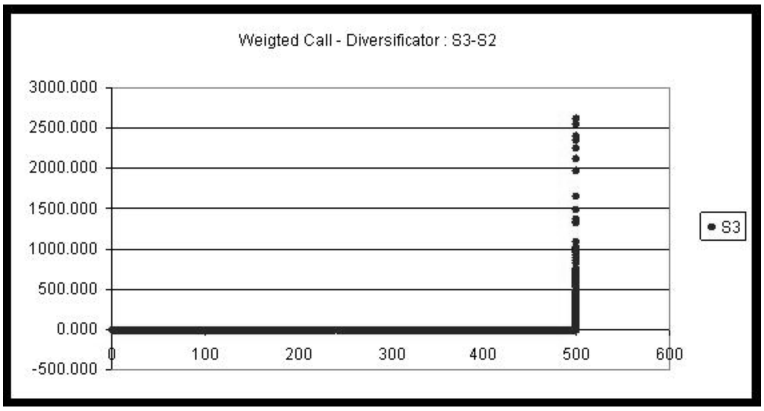

Figure 10: Strat 2 : Call Weigted

5.8.2 Strat 3 and 2

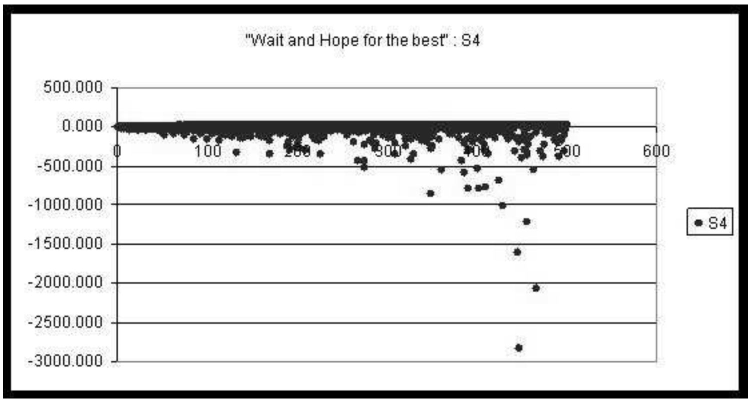

Figure 11: Strat 4: Wait and Hope

5.8.3 Strat 4

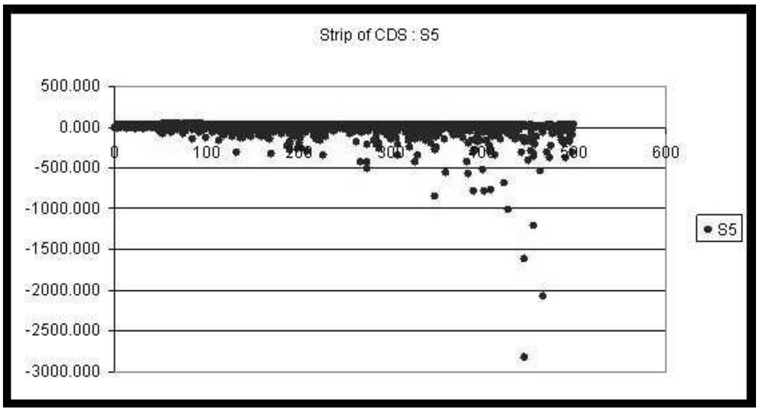

Figure 12: Strat 5: Strip of CDS spread

5.8.4 Strat 5

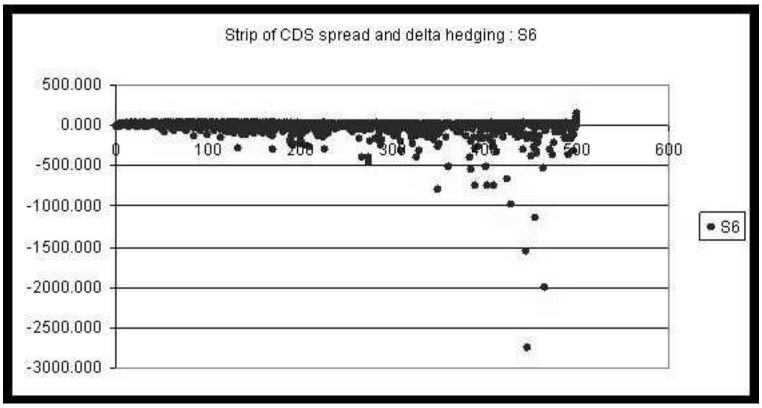

Figure 13: Strat 6: Strip of CDS spread and Delta Hedging

5.8.5 Strat 6

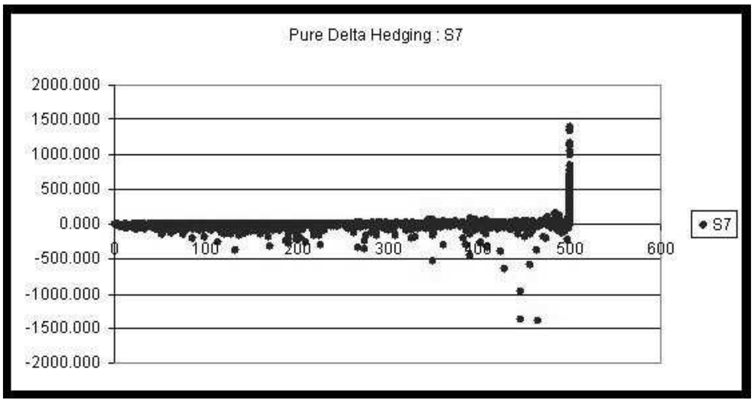

Figure 14: Strat 7: Pure Dynamic Hedging

5.8.6 Strat 7

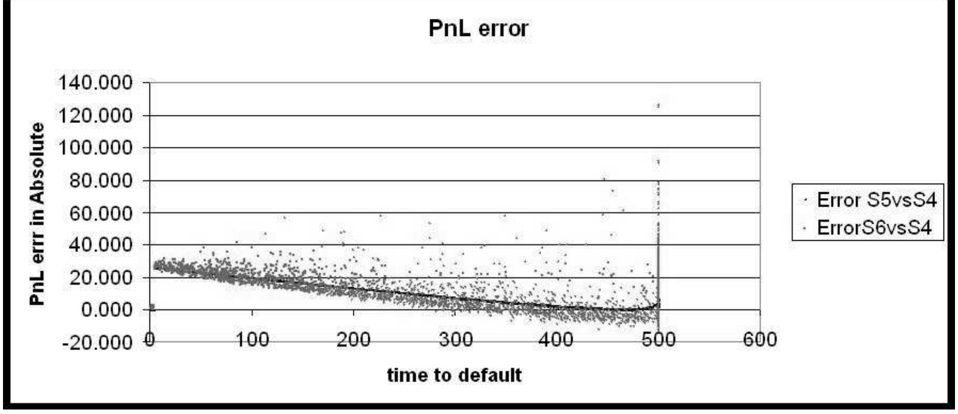

Figure 15: Strat 7: Pure Dynamic Hedging

5.8.7 Error S4 against S5 and S6