CVA Implied Vol and Netting Arbitrage Introduction

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First version: Nov 20, 2010
This version: Jun 3, 2011

Abstract

After Lehman default (credit crisis which started in 2007), practitioners considered the default risk as a major risk. The Industry began to charge for the default risk of any derivatives. In this article we try to extend the work of V.Piterbarg who established the fundamental of a new world in the pricing of derivatives. Our main focus will be on the Equity CVA but can be extended to any asset class. In this article we established the default risky price of particular space of derivatives based on vanilla CVA then we introduced the CVA implied Volatility and described a new pricing methodology. It is the first time that the CVA premium is not consider as a binaire relation, in this paper we established the link and arbitrage opportunities related to the derivative hedge portfolio and the CVA premium.

*The opinions of this article are those of the author and do not reflect in any way the views or business of his employer.

Keywords: CVA, netting, arbitrage, CVA implied volatility, CVA local volatility.

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1 Introduction

After Lehman Brothers Default, the financial industry started treating the counterparty default risk as a major risk. The curve spread ceased to be negligible, and the use of collateral agreements and CVA charges became very standard as protection against a default (see S.Alavian et al [5], for more details).

No one is too big too default, everybody owns a default risk.

In this context, how does this fear affect the derivatives pricing? How can it be consistent with the usual risk neutral pricing theory?

In [4], V.Piterbarg managed to elaborate one building block of the default fear pricing theory. Focus on the discounting, he talked about funding and collateral at the derivative level. He linked the intuition and the pricing theory although these terms are defined at the counterparty level. In the same way, we would like to establish a relation between a derivative and its vanilla hedging portfolio in a default risky world.

The industry started to consider the CVA for each derivative given by these well known formulae:

$$CVA^{\Pi} \quad = \quad \mathbb{E}^{\mathbb{Q}}\left[\int_{t_0}^{T}(1-R)\, e^{-\int_{t_0}^{t} r_s ds}\Pi_t d\mathbb{Q}\left(\tau < t\right)\right]$$

The question concerning the consideration of the CVA as a premium will be a topic of a subsequent publication. In this article, we will assume that it is an effective price (eg there is a strategy able to replicate the potential loss at default).

After describing a set of adequate derivatives, we will elaborate two types of arbitrage opportunities. The first one with the CVA Implied Volatility Introduction, the second based on netting arbitrage.

The recent industry developments have been focus on the CVA for interest derivatives and credit derivative portfolios (see for instance Brigo [1], Elouerkhaoui [2], M.Jeanblanc [6]), however in ISDA half 2008, the Equity Commodity and FX market were higher in volume and exposure than the Credit one. Therefore we will be focus on the Equity market but our framework and results can be extended to any other asset classes.

Our conclusions are the followings; as market participant we can not consider the CVA as a One credit spread related product. The CVA as a tradable asset should be computed in regards of our hedging strategy. If the structure composed by a short derivative position and a long its CVA is not hedge able, it doesnt mean the structure is not arbitrage able.

For a derivative P, we define the default risky price $\bar{P}$

$$\begin{aligned}\bar{P}\left(t_0, T\right) &= P\left(t_0, T\right)\\ &- CVA^{P(t_0,T)}\end{aligned}$$

Our work represents a big interest in sense that it takes into account for the first time the netting arbitrage in the computation of the CVA, the CVA is no longer an unidirectional risk (in fact it has never been)! We defined the CVA Implied Volatility to highlight trade pricing inconsistencies. In addition our CVA Implied volatility representation allows us to speed up the computation of the Risky Price of derivatives (given the computation of CVA risky vanillas prices).

From the pre-Lehman pricing theory a price is associated to a hedging strategy which can be presented as follows

$$\begin{aligned} P(t,T) &= \Pi_t \\ &= \sum_{i=1}^{N} w_t^i Call_t^i \left(T_{\eta(i)}, K_{\gamma(i)}\right) \\ &+ w_t^0 S_t \end{aligned}$$

where w_i are stochastic describing a hedging strategy. If we restrict ourselves to the positive derivatives with constant allocation (eg $sign\left(w_t^i\right)$ is constant for all t)

we have :

$$\begin{aligned} CVA^{P(t,T)} &= CVA^{\Pi_t} \\ &= \sum_{i=0}^{N} w_t^i CVA^{Call_t^i\left(T_{\eta(i)}, K_{\gamma(i)}\right)} \end{aligned}$$

$\sum_{i=0}^{N} w_t^i Call_t^i \left(T_{\eta(i)}, K_{\gamma(i)}\right)$ will be called the hedging representation of P.

2 Modelling CVA arbitrage

First we will introduced the CVA implied volatility, it is a volatility object $\tilde{\sigma}(T,K)$ such as :

$$\begin{aligned} \bar{C}all\,(T,K;R,\lambda) &= Call^{mkt}(T,K) - CVA^{Call(T,K)} \\ &= \underbrace{\left[1-(2-R)\,\mathbb{Q}\,(\tau<T)\right]}_{\alpha(R,Pd)} Call^{BS}\left(T,K,\tilde{\sigma}(T,K)\right) \end{aligned}$$

with R the recovery of the counterparty and λ representing the probability of default term structure.

We have the same type of arbitrage free relationship for the risky price volatility surface and the classical one (pre-Lehman); Call Spread, Callendar and Butterfly arbitrage.

In this section, we will consider a Mirror World (eg a world or subset of market participant with the same default term structure).

In this context, we image a counterparty B which is kind to buy from us a spread of two risky calls $\bar{C}all\,(T, K_2) - \bar{C}all\,(T, K_1) > 0$ with $K_1 < K_2$. We will take a position short on this spread.

In case of default of B, in one hand we will receive from the administrators the recovery of the $RCall_{\tau^B}\,(T, K_1)$ and our CVA desk will give us $(1 - R)\,Call_{\tau^B}\,(T, K_1)$. In other hand, we will owe $Call_{\tau^B}\,(T, K_2)$. The balance is positive resulting of the classical Call Spread arbitrage.

If we default, we will receive $Call_{\tau^{Me}}\,(T, K_1)$ from B and give to B via the admistration process $RCall_{\tau^{M}e}\,(T, K_2)$. The balance is still positive.

At default, the payoffs implies a positive balance. Therefore, there is a clear arbitrage in sense where B paid for an option which is always against him whatever the event. A similar demonstration could be done for the calendar and butterfly.

We also want to notify that these arbitrage relationships are still valid for $R^{Me} > R^B$ and $\lambda^{Me} > \lambda^B$ (the last ineguality means that the default probability term structure of B is higher than ours).

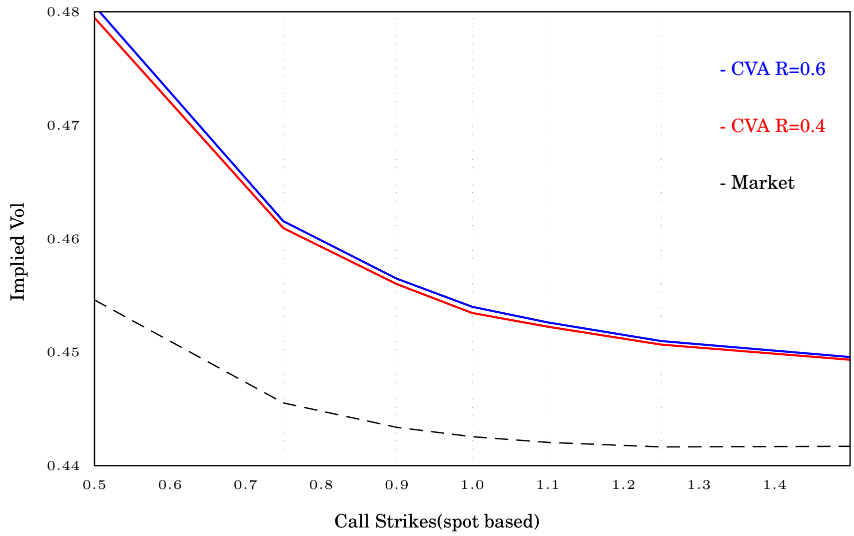

Figure 1: 10Y Implied CVA, Recovery

In the Figure 1 we plot the CVA smile as a function of different recovery values.

More generally thank to our CVA Implied vol representation we would like to be able to detect quickly the CVA modelling arbitrage resulting of inconsistencies between the risky price of derivative against its vanilla hegding representation.

In a specific set of derivatives, the positive derivative statically replicable (eg $w_t^i = w_0^i$), we

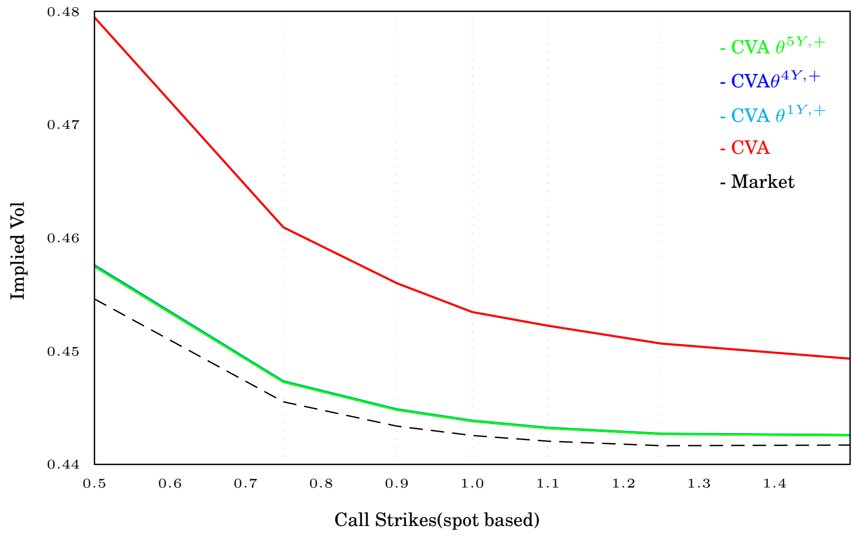

Figure 2: 10Y Implied Vol CVA, 0.001% stress on the Pds

can very quickly linked the derivative and its hedging representation default risky price.

$$\begin{aligned} \bar{P}(t_0, T) &= \bar{\Pi}_{t_0} \\ &= \sum_{i=0}^{N} w_0^i \bar{C}all_{t_0}\left(T_{\eta(i)}, K_{\gamma(i)}\right) \end{aligned}$$

given our CVA implied Volatility, we could easely see from a risk price of a derivative which belongs to the set of the statically replicable an arbitrage opportunity with his hedging representation.

3 Netting Arbitrage

In this section, we will consider a third party D as hedge seller. We will construct the same type of portfolio relaxing our Mirror context hypothesis.

It is clear from the derivative hedging representation that the long vanilla positions will induce sensitivity to the credit spread of D in our hedging cost (see shema in Appendix).

As a derivative seller, we can not allow the buyer derivative to have any netting benefit (because this benefit will cost me a certain loss). The CVA can not be a function of only our potential default, indeed as seller the diversification of our hedging portfolio exposed us to others default entities. We have to transfer this risk back to the derivative buyer.

We will consider a simple and concrete example, the sell of a digital risky price. A digital can be replicated statistically via a call spread position. As explained Kamtchueng in [3], it is not a vanilla option .

Probabilities Default			$Digital$	$CallSpread$	
Pd^{Me}	Pd^{B}	Pd^{D}	CVA	CVA	CVA_{NN}
θ	θ	θ	0.01205	0.01460	0.01460
θ^-	θ	θ	0.01205	0.01460	0.01548
θ^+	θ	θ	0.01205	0.01460	0.01373
θ^+	θ	θ^+	0.01205	0.01460	0.01502
θ^+	θ	θ^-	0.01205	0.01460	0.01244
θ	θ	θ^-	0.01205	0.01460	0.01331
θ	θ	θ^+	0.01205	0.01460	0.01590

Table 1: Digital and Call Spread 2Y Netting Impact, Notional 100

In order to deliver the digital by taking a static position on a call spread, we can not accept to have a CVA charge superior to the one we will pay for our hedging portfolio position. Without communication between desks, we will have an arbitrable structure even for a simple payoff.

In the table above, we consider a counterparty B which would like to buy a Digital, we keep its credit term structure constant. the first two column describe the price of the digitale and call spread in a Mirror world in which B belongs. The last column describes the change on the CVA as function of the change on the probability of default term structure. It is clear that B for some cases can sell the Call Spread to someone, therefore it will be long CVA ($CVA = 0.0146$), then it will buy the hedge representation protfolio which a lower charge. So B will have make a profit as explained in the shema in Appendix.

4 Conclusion

We have established for first time the hedging cost of specific Risky derivatives. Indeed as a tradable asset, our hedging strategy involves our credit spread but also the ones of our hedge sellers.

This concept is as important as the modelling of the CVA itself (which involves forward market dynamic and intensive consistent multi factor simulator factory). The communication between traders and quants are essentials, as described V.Piterbarg some trading choices will impact the pricing. Not only on the discount part but also on our sensitivities as we shown with the Digital option example.

Our view of the CVA as a premium will be discussed in a subsequent publication.

Our work can be extended to more exotic products via the semi-static replication completeness.

Future works will expose a more ambitious use of CVA implied volatility : for dynamic hedging purposes via a calibration of the CVA implied Vol or the introduction of the CVA local volatility.

Theses methods speed up the time computation of the risky price given the risky price of the vanillas (which can be done ingeniously in one Monte Carlo simulation given some classical assumptions for the CVA)

One of the major issue in regard to counterparty risk is the wrong-way risk. The consideration of the hedge seller credit rating, could be seen as a second order wrong-way risk. Indeed it could be defined by the correlation between the credit quality of the counterparty and the one of the hedging seller.

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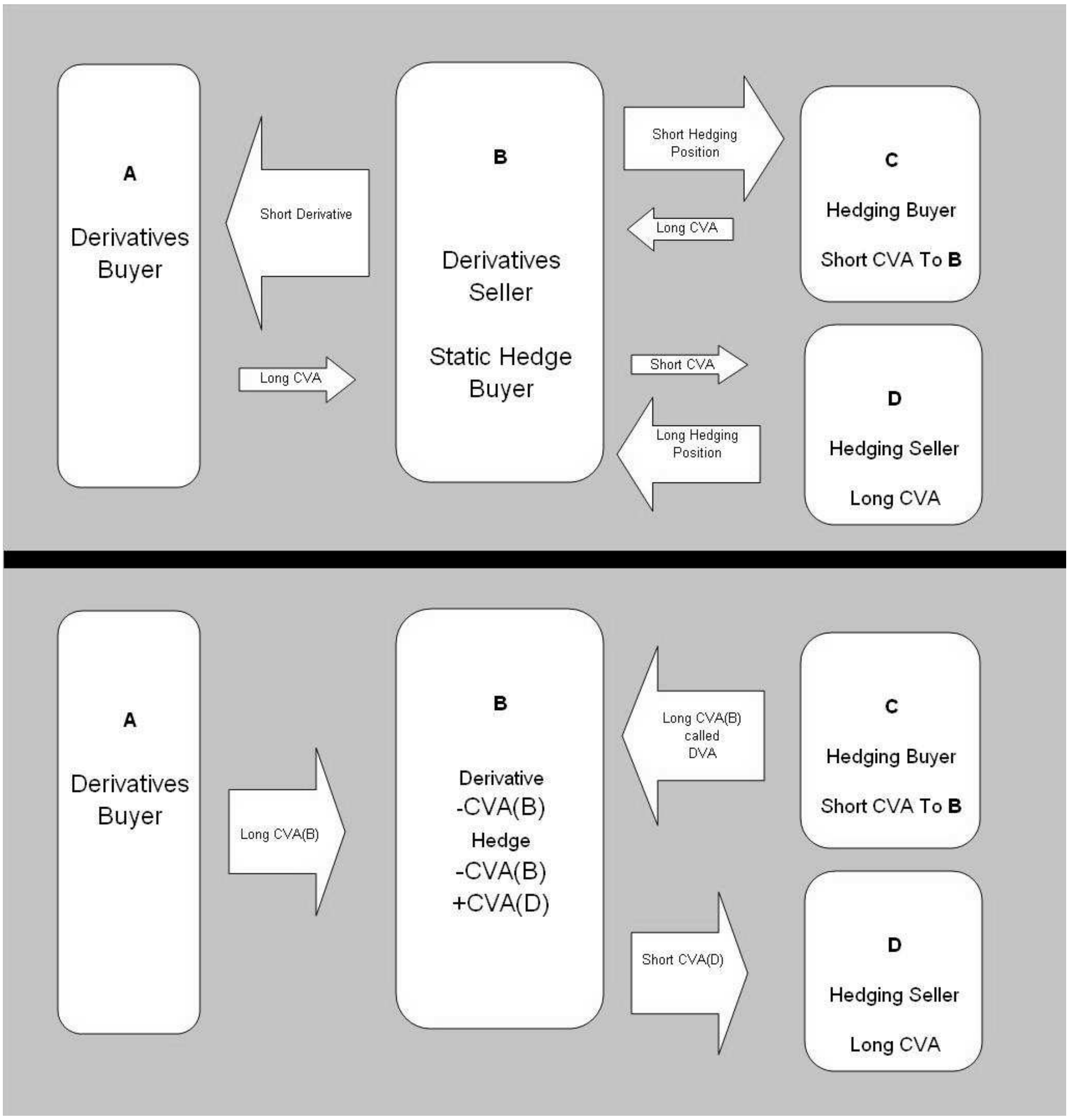
5 Numerical Test Notations

- CVA^{NN} credit valuation adjustment without netting agreement
- $\alpha\left(R, Pd\right)$ or if there is no ambiguity α is the CVA implied factor, it will be defined later
- Pd^{X} the credit spread market term structure of X (hedging seller)
- Pd^{Me} our credit spread market term structure as a entity (derivative seller)
- Pd^{Cy} the credit spread market term structure of Cy the counterparty (derivative buyer)
- $\theta := \left(\mathbb{Q}\left(\tau < t_i\right)\right)_{i=1\ldots N}$ a credit spread market term structure which would be precise on Appendice
- $\theta^{*} := \mathbb{Q}\left(\tau < t_i\right) * \epsilon$ with $* \in \{-, +, \pm, \mp\}$ the term structure is shifted by the absolute value $\epsilon = 0.00001$.
  - If $* = \pm$ then $\mathbb{Q}\left(\tau < t_i\right) + \epsilon, \forall i < \frac{N}{2}$ and $\mathbb{Q}\left(\tau < t_i\right) - \epsilon, \forall i > \frac{N}{2}$
  - If $* = \mp$ then $\mathbb{Q}\left(\tau < t_i\right) - \epsilon, \forall i < \frac{N}{2}$ and $\mathbb{Q}\left(\tau < t_i\right) + \epsilon, \forall i > \frac{N}{2}$
- $\theta^{iY,*}$ a credit spread market term structure which correspond to $\forall t_i > iY, \mathbb{Q}\left(\tau < t_i\right) * \epsilon$
- $\theta^{*,*} := \mathbb{Q}\left(\tau < t_i\right) * \epsilon * \epsilon$ with $* \in \{-, +, \pm, \mp\}$ the term structure is shifted by the absolute value $\epsilon = 0.00001$
- $d\mathbb{Q}\left(\tau < dt\right) := \mathbb{Q}\left(\tau \in t \pm dt\right) dt$

6 Schema Netting Arbitrage



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