

Corrections (2)

▷ **Exercice 2.** Résoudre les systèmes suivants :

$$(S_1) : \begin{cases} 2x - \sqrt{3}y = 0 \\ \sqrt{3}x - 3y = -1 \end{cases} \quad (S_2) : \begin{cases} \frac{1}{10}x - \frac{1}{20}y = 1 \\ \frac{2}{5}x - \frac{1}{10}y = 10 \end{cases} \quad (S_3) : \begin{cases} \frac{5}{3}x - \frac{1}{4}y = \frac{35}{8} \\ \frac{1}{3}x - \frac{1}{20}y = \frac{7}{8} \end{cases} \quad (S_4) : \begin{cases} \frac{2}{x+1} - \frac{5}{y-2} = -4 \\ \frac{3}{x+1} + \frac{2}{y-2} = 13 \end{cases} \quad (S_5) : \begin{cases} 3x^2 - y^2 = 3 \\ x^2 + 2y^2 = 22 \end{cases}$$

On notera $d = ab' - a'b$ le déterminant d'un système.

• $(S_1) : d = 2 \times (-3) - \sqrt{3} \times (-\sqrt{3}) = -6 + 3 = -3 \neq 0$

$$(S_1) : \begin{cases} 2x - \sqrt{3}y = 0 \\ \sqrt{3}x - 3y = -1 \end{cases} \quad (\times \sqrt{3}) \iff \begin{cases} 2\sqrt{3}x - 3y = 0 \\ \sqrt{3}x - 3y = -1 \end{cases} \quad (L_1 \leftarrow L_1 - L_2) \iff \begin{cases} 2\sqrt{3}x - 3y - \sqrt{3}x + 3y = 1 \\ \sqrt{3}x - 3y = -1 \end{cases} \iff \begin{cases} \sqrt{3}x = 1 \\ \sqrt{3}x - 3y = -1 \end{cases}$$

$$\iff \begin{cases} x = \frac{1}{\sqrt{3}} \\ \sqrt{3}x - 3y = -1 \end{cases} \iff \begin{cases} x = \frac{1}{\sqrt{3}} \\ \sqrt{3} \times \frac{1}{\sqrt{3}} - 3y = -1 \end{cases} \iff \begin{cases} x = \frac{1}{\sqrt{3}} \\ 1 - 3y = -1 \end{cases} \iff \begin{cases} x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \\ y = \frac{2}{3} \end{cases} \quad \text{donc le système admet un}$$

unique couple solution : $(x; y) = \left(\frac{\sqrt{3}}{3}; \frac{2}{3} \right)$

• $(S_2) : d = \frac{1}{10} \times \left(-\frac{1}{10}\right) - \frac{2}{5} \times \left(-\frac{1}{20}\right) = 0,01 \neq 0$

$$(S_2) : \begin{cases} \frac{1}{10}x - \frac{1}{20}y = 1 \\ \frac{2}{5}x - \frac{1}{10}y = 10 \end{cases} \quad (\times 2) \iff \begin{cases} \frac{1}{5}x - \frac{1}{10}y = 2 \\ \frac{2}{5}x - \frac{1}{10}y = 10 \end{cases} \quad (L_1 \leftarrow L_1 - L_2) \iff \begin{cases} -\frac{1}{5}x = -8 \\ \frac{2}{5}x - \frac{1}{10}y = 10 \end{cases} \iff \begin{cases} x = 8 \times \frac{5}{1} = 40 \\ \frac{2}{5} \times 40 - \frac{1}{10}y = 10 \end{cases}$$

$$\iff \begin{cases} x = 40 \\ y = 60 \end{cases} \quad \text{donc le système admet un unique couple solution : } (x; y) = (40; 60)$$