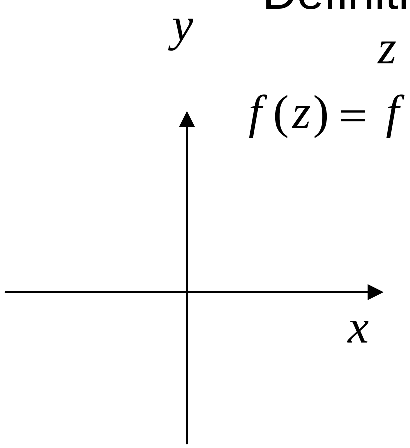
Liens solutions équation de Laplace
Fonctions analytiques

Définitions



$$z = x + i.y$$

$$f(z) = f(x,y) = u(x,y) + i.v(x,y)$$

f est analytique

$\Leftrightarrow$ dérivée suivant Ox = dérivée suivant Oy

$$f'(z) = \frac{f(x+dx,y) - f(x,y)}{dx} = \frac{f(x,y+dy) - f(x,y)}{i.dy}$$

$$\Leftrightarrow \frac{\partial u}{\partial x}(x,y) + i.\frac{\partial v}{\partial x}(x,y) = -i.\frac{\partial u}{\partial y}(x,y) + \frac{\partial v}{\partial y}(x,y)$$

Ou encore

$$\Rightarrow \begin{cases} \frac{\partial u}{\partial x}(x,y) = \frac{\partial v}{\partial y}(x,y) \\ \frac{\partial v}{\partial x}(x,y) = -\frac{\partial u}{\partial y}(x,y) \end{cases}$$

$$\begin{matrix} \times \frac{\partial}{\partial x} \\ \times \frac{\partial}{\partial y} \end{matrix} \Rightarrow \begin{cases} \frac{\partial^2 u}{\partial x^2} + i.\frac{\partial^2 v}{\partial x^2} = -i.\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x \partial y} \\ \frac{\partial^2 u}{\partial x \partial y} + i.\frac{\partial^2 v}{\partial x \partial y} = -i.\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y^2} \end{cases}$$

$\longrightarrow$

$$\frac{\partial^2 u}{\partial x^2} + i.\frac{\partial^2 v}{\partial x^2} = -i.\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x \partial y}$$

x (-i) $\longrightarrow$

$$-i.\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial^2 u}{\partial y^2} - i.\frac{\partial^2 v}{\partial y^2}$$

somme

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + i.(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) = 0 + i.0$$

$$\frac{\partial^2 f}{\partial x^2} = 0$$

$$\Rightarrow \boxed{\begin{matrix} \Delta f = 0 \\ \Delta u = 0 \\ \Delta v = 0 \end{matrix}}$$